

Definitions

convergent sequence:

convergent series:

alternating series:

absolutely convergent:

conditionally convergent:

radius of convergence:

interval of convergence:

Taylor Series:

Maclaurin Series:

Example of How to Use the Test

Hypotheses

Test

Divergence

Integral

Comparison

Limit Comparison

Alternating Series

Ratio

Root

Divergence

$$|r| \geq 1$$

Convergence

$$|r| < 1$$

Expansion

$$a + ar + ar^2 + ar^3 + \dots$$

Series

$$\sum_{n=0}^{\infty} ar^{n-1}$$

Geometric

p-series

harmonic

power

alternating

Taylor

Maclaurin

e^x

$\sin x$

$\cos x$

$(1+x)^k$

Questions

- ① Suppose $\sum a_n = 3$ and s_n is the n^{th} partial sum of the series.
What is $\lim_{n \rightarrow \infty} a_n$?
What is $\lim_{n \rightarrow \infty} s_n$?
- ② Suppose $f(x)$ is the sum of a power series with radius of convergence R .
How do you differentiate $f(x)$?
What is the radius of convergence of $f'(x)$?
How do you integrate $f(x)$?
What is the radius of convergence of $\int f(x) dx$?
- ③ a) Write an expression for the n^{th} -degree Taylor polynomial of f centered at a .
b) Write an expression for the Taylor series of f centered at a .
c) Write an expression for the Maclaurin series of f .
d) How do you show that $f(x)$ is equal to the sum of the Taylor series?
e) State Taylor's Inequality

True / False

- ① If $0 \leq a_n \leq b_n$ and $\sum b_n$ diverges then $\sum a_n$ diverges
- ② If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ is convergent.
- ③ If $\sum c_n b^n$ is convergent then $\sum c_n (-b)^n$ is convergent
- ④ If $\lim_{n \rightarrow \infty} a_n = L$, then $\lim_{n \rightarrow \infty} a_{2n+1} = L$.
- ⑤ If $-1 < a < 1$, then $\lim_{n \rightarrow \infty} a^n = 0$
- ⑥ If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent.
- ⑦ If $a_n > 0$ and $\sum a_n$ converges, then $\sum (-1)^n a_n$ converges

Problems

Determine convergence or divergence

$$\textcircled{1} \sum_{n=1}^{\infty} \frac{n}{n^3+1}$$

$$\textcircled{2} \sum_{n=1}^{\infty} \frac{n^3}{5^n}$$

$$\textcircled{3} \sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$$

$$\textcircled{4} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$$

$$\textcircled{5} \sum_{n=1}^{\infty} (-1)^{n-1} n^{-1/3}$$

$$\textcircled{6} \sum_{n=1}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$$